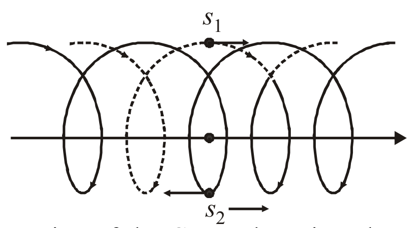
6 System of Particles and Rotational Motion

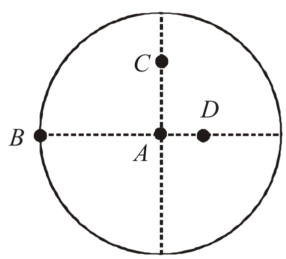
Diagram Based Questions :

1. The motion of binary stars , S_1 and S_2 is the combination of ....*X*.... and ....*Y*.... . Here, *X* and *Y* refer to



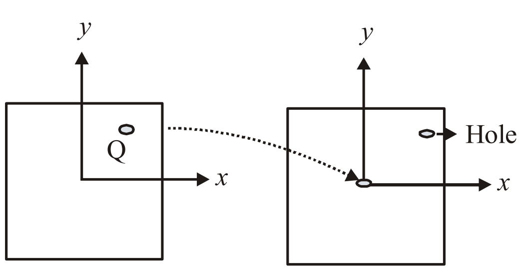
(a) motion of the CM and motion about the CM
(b) motion about the CM and motion of one star
(c) position of the CM and motion of the CM
(d) motion about CM and position of one star

2. The moment of inertia of a uniform circular disc (figure) is maximum about an axis perpendicular to the disc and passing through



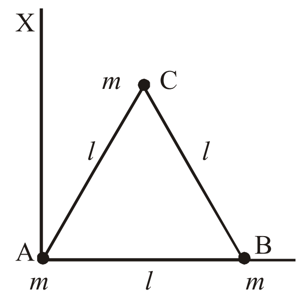
(a) *B* (b) *C*
(c) *D* (d) *A*

3. A uniform square plate has a small piece Q of an irregular shape removed and glued to the centre of the plate leaving a hole behind. Then the moment of inertia about the z-axis



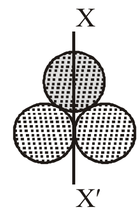
(a) increases
(b) decreases
(c) remains same
(d) changed in unpredicted manner.

4. Three particles, each of mass *m* gram, are situated at the vertices of an equilateral triangle ABC of side ℓ cm (as shown in the figure). The moment of inertia of the system about a line AX perpendicular to AB and in the plane of ABC, in gram-cm^2 units will be



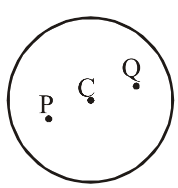
(a) $\frac{3}{2}m\ell^2$
(b) $\frac{3}{4}m\ell^2$
(c) $2\,m\ell^2$
(d) $\frac{5}{4}m\ell^2$

5. Three identical spherical shells, each of mass m and radius r are placed as shown in figure. Consider an axis XX' which is touching to two shells and passing through diameter of third shell. Moment of inertia of the system consisting of these three spherical shells about XX' axis is



(a) $3mr^2$
(b) $\frac{16}{5}mr^2$
(c) $4mr^2$
(d) $\frac{11}{5}mr^2$

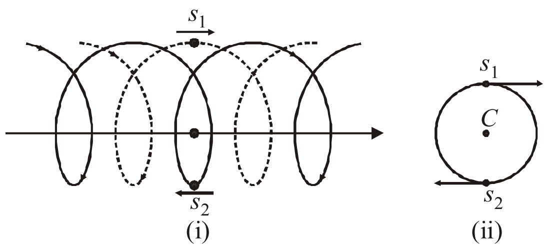
6. A disc is rolling (without slipping) on a horizontal surface C is its centre and Q and P are two points equidistant from C. Let V_p, V_q and V_c be the magnitude of velocities of points P, Q and C respectively, then



(a) $V_Q > V_C > V_P$
(b) $V_Q < V_C < V_P$
(c) $V_Q = V_P,\ V_C = \frac{1}{2}V_P$
(d) $V_Q < V_C > V_P$

Solution

1. **(a)** When no external force acts on the binary star, its CM will move like a free particle [Fig. **(a)**]. From the CM frame, the two stars will seems to move in a circle about the CM with diametrically opposite positions.



(i) Trajectories of two stars. S_1 (dotted line) and S_2 (solid line) forming a binary system with their centre of mass C in uniform motion

(ii) The same binary system, with the centre of mass C at rest.

So, to understand the motion of a complicated system, we can separate the motion of the system into two parts. So, the combination of the motion of the CM and motion about the CM could described the motion of the system.

2. **(a)** According to parallel axis theorem of the moment of Inertia

$I = I_{cm} + md^2$

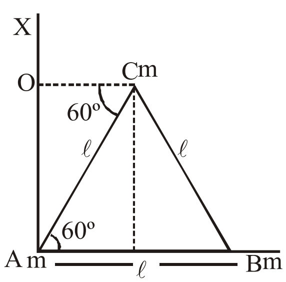
d is maximum for point B so I_{max} about B.

3. **(b)**

4. **(d)** $I_{AX} = m(AB)^2 + m(OC)^2$

$= m\ell^2 + m(\ell \cos 60^\circ)^2$

$= m\ell^2 + m\ell^2/4 = 5/4\, m\ell^2$

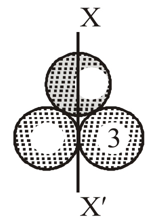


5. **(c)** Moment of inertia of shell 1 along diameter

$$I_{diameter} = \frac{2}{3}MR^2$$

Moment of inertia of shell 2 = m. i of shell 3

$$= I_{tengential} = \frac{2}{3}MR^2 + MR^2 = \frac{5}{3}MR^2$$



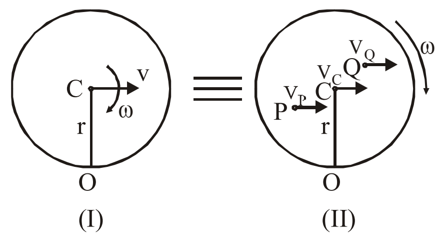
So, I of the system along x x[1]

$= I_{diameter} + (I_{tengential}) \times 2$

or, $$I_{total} = \frac{2}{3}MR^2 + \left(\frac{5}{3}MR^2\right) \times 2$$

$$= \frac{12}{3}MR^2 = 4MR^2$$

6. **(a)**



From Fig. (I), we have OC = r (radius)

Therefore, $v = r\omega$

Since , ω = constant, therefore $v \propto r$

Now, form Fig (II), it is clear that the distance, $OP < OC < OQ \Rightarrow V_P < V_C < V_Q$ or $V_Q > V_C > V_P$.